COMPUTATIONAL IMAGING AND VISION

Geometry-Driven Diffusion in Computer Vision

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Preface

Scale is a concept the antiquity of which can hardly be traced. Certainly the familiar phenomena that accompany scale changes in optical patterns are mentioned in the earliest written records. The most obvious topological changes such as the *creation* or *annihilation* of details have been a topic of fascination to philosophers, artists and later scientists. This appears to be the case for all cultures from which extensive written records exist. For instance, chinese 17^{th}c artist manuals remark that "distant faces have no eyes". The *merging* of details is also obvious to many authors, *e.g.*, Lucretius mentions the fact that distant islands look like a single one. The one topological event that is (to the best of my knowledge) mentioned only late (by John Ruskin in his "Elements of drawing" of the mid 19^{th}c) is the *splitting* of a blob on blurring. The change of images on a gradual increase of resolution has been a recurring theme in the arts (*e.g.*, the poetic description of the distant armada in Calderón's *The Constant Prince*) and this "mystery" (as Ruskin calls it) is constantly exploited by painters.

It was facts as these that induced me to attempt a mathematical study of scale, especially when my empirical researches in the psychophysics of spatial contrast in the peripheral visual field of human observers convinced me that the brain samples and represents the optic array at many scales simultaneously. Events accelerated around 1980. I remember that I met Andy Witkin at the 1^{st} David Marr memorial workshop thrown by Whitman Richards at Cold Spring Harbor: I believe that there I heard the term "scale-space" for the 1^{st} time. By then I had set up the diffusion equation formulation that connects the scales formally though I didn't publish because the whole thing still had too much ad hockery to it. However, shortly thereafter the "causality constraint" occurred to me (triggered by a fascination with cartographic "generalization") and I saw the pieces click together except for a few frayed ends. (Although the notion of "causality" in the scale domain remains indeed a corner stone of the theory in my opinion, its original formulation—though correct—was perhaps unfortunate because often misunderstood even to the point were people believe it to be false.) Since then much has happened and I feel that today "scale-space" can stand on its own.

"Pure scale-space" can be constructed from a few basic assumptions that essentially express *total a priori ignorance.* Thus no scale, place or orientation is potentially special in any way. The resulting pristine structure has the beauty and fascination of something inevitable, a *discovery* rather than a mere *construction.* However, it tends to leave practical people unsatisfied because in any *real* application one knows many things that might advantageously be exploited in the analysis. This might be any form of prior knowledge, including rather abstract notions of what structure to expect, so that the actual data can be used to control the operations performed on themselves. It seems natural to try to use the freedom left in pure scale-space to attach such control handles and thereby turn the nymph into a handmaiden. It is perhaps fair to say that no one today knows how to proceed in an apparently *necessary* way here, thus the theory sprouts into a multitude of complementary and concurrent approaches.

I have read through the manuscript of this book in fascination. Most of the approaches that have been explored to tweak scale-space into practical tools are represented here. Many of the contributions have a certain attraction of their own and several appear to be promising in future applications. It is easy to appreciate how both the purist and the engineer find problems of great interest in this area. The book is certainly unique in its scope and has appeared at a time where this field is booming and newcomers can still potentially leave their imprint on the core corpus of scale related methods that will slowly emerge. As such the book is a very timely one. It is quite evident that it would be out of the question to compile anything like a textbook at this stage: This book is a snapshot of the field that manages to capture its current state very well and in a most lively fashion. I can heartily recommend its reading to anyone interested in the issues of image structure, scale and resolution.

One thing that the reader will notice is the nature of the pictures that go to illustrate many of the contributions: They are quite distinct in character from the type of illustration one meets in the pure diffusion literature. The difference lies in the very sharpness of the "blurred" images. Here we meet with the same fascination one finds in *e.g.*, Canaletto's figures in his paintings of Venice's squares: As the viewer looks at the painted figures farther and farther away in the perspective of the pavement details are lost though the figures are always made up of sharply delimited blobs of paint. In the near foreground a single blob may stand for the button of a waistcoat, in the far distance a whole head, yet everything appears mysteriously "sharp" at *any* distance. It is as with cartographic generalization where the general shape of a city may suddenly give way to a circular disc.

For those readers sensitive to such matters the mathematics also has a quite different flavor from the original diffusion literature. We meet with an *embarras de richesse* perhaps typical of a field in rapid development. It seems almost inevitable that many of the various strands will eventually come together in the weave of something novel.

I compliment the editor on this timely and fascinating book and promise the reader pleasure and profit.

Jan J. Koenderink, Utrecht University

Foreword

This book is the fruit of a European-American transatlantic collaboration, sponsored by an ESPRIT–NSF grant (travel and subsistence) awarded for the period 1993–1996 [96]. The book brings together the important groups active in this field, many of which have an outstanding record of achievements. The group formed in 1992, with a collaborating effort to write the grant proposal. Since then a number of successful meetings were held: In May 1993 the kick-off meeting was held in Berlin. Here the concept of the book was born. The next meeting was a joint meeting with the Newton Mathematical Institute, end November 1993. The workshop in Stockholm, May 1994, in connection with the 3rd European Conference on Computer Vision saw the final draft presentation of this book as first tangible result of the collaboration.

A short synopsis of the book:

Chps. 1 and 2 by Lindeberg and ter Haar Romeny give a tutorial introduction to linear scale-space. It includes the history, its (axiomatic) foundations, basic theory and an overview of the current research topics, related kernels such as wavelets and Gabor functions, and it introduces the notion of scaled differential operators and their applications. Directions into the study of the 'deep structure' of scale-space are indicated.
From Chp. 3 onwards the book focuses on non-linear or geometry-driven diffusion. Coarsely, two major approaches can be discriminated, each with their variations: the variational approach, were the energy of a functional is minimised, the functional being some cost function that can be manipulated, and on the other hand the nonlinear PDE approach, where the evolution of the image is expressed as some function of invariant properties of the image. Historically geometry-driven diffusion was introduced as being some local function of edge-strength. This is discussed in Chp. 3 by Perona, Shiota and Malik, together with a critical analysis of the resulting non-linear PDE and the discrete maximum principle. In Chp. 4 by Whitaker and Gerig not only edges are involved, but a complete 'feature space' gives rise to multi-valued or vector-valued diffusion, using both (higher order) features in a

geometry-limited diffusion scheme, as local frequency decomposition in a spectra-limited diffusion scheme.
The variational approach is given a firm Bayesian basis by Mumford, one of its original authors, in Chp. 5. Four probabilistic models are presented from reasoning and axioms, and shown on examples.
The variational approach is a prototypical example of a 'free-discontinuity problem'. Critical questions like uniqueness of the solution and existence of an optimal segmentation are raised by Leaci and Solimini in Chp. 6.
Edges get displaced when they are blurred. Nordström in Chp. 7 studies edges as finite unions of smooth curves (line drawings) and demonstrates the existence of a solution in this particular case.
In the next two chapter a link is being established between the variational approach and the PDE-based evolutions. In Chp. 8 Richardson and Mitter extend the variational approach with the theory of Γ-convergence, i.e. the approximation of one functional by another. Replacing the edge set with a smooth function then leads to edge detection by *coupled* partial differential equations. These are further discussed and elaborated in Chp. 9 by Proesmans, Pauwels and van Gool, extending the applications to second order smoothing, multispectral images, optic flow and stereo.
An attempt to some unification is presented in Chp. 10 by Alvarez and Morel, who summarize the morphological approach to geometry-driven diffusion in the light of many other 'scale-space' or multi-scale approaches. The authors establish a number of basic principles for the visual pyramid and derive the 'fundamental equation' of shape analysis.
The general form of the nonlinear diffusion equations are always some function of a particular invariant. In Chp. 11 Olver, Sapiro and Tannenbaum first give a tutorial on basic invariant theory, starting from Lie groups and prolongations. Images can be considered embedded sets of curves. The authors then describe in detail the invariant *curve evolution* flows under Euclidean, similarity, affine and projective transformations, as well as the impact of certain conservation laws.
Kimia, Tannenbaum and Zucker take methods from optimal control theory in Chp. 12. Dynamic programming and the application of the Hamilton-Jacobi equation are applied to shape theory, morphology, optic flow, nonlinear scale-space and shape-from-shading.
Two chapters introduce and apply the notion of the covariant formalism for coordinate free reasoning. Florack et al. study in Chp. 13 a special nonlinear scale-space, which can be mapped onto a linear scale-space. The Perona and Malik equation is studied in this context, and a log-polar foveal scale-space is presented as one of the examples. Eberly in Chp. 14 gives detailed examples of nonlinear scale-space properties once a proper *metric* is chosen. The appendix of Chp. 13 is a tutorial for the covariant formalism.

The volume ends with a chapter on implementations. They can be found scattered in the book, but here Niessen et al. show the feasibility of many nonlinear PDE approaches by implementing them in a forward Euler fashion with Gaussian derivatives on a variety of (medical) images.

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Bart M. ter Haar Romeny, Utrecht, June 1994

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